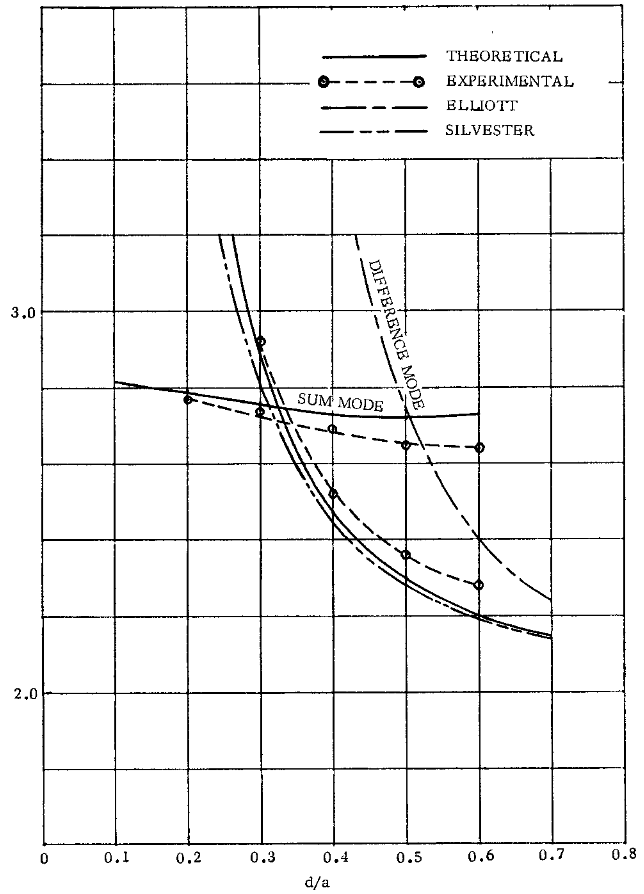

Fig. 4. Comparison of results.

and Elliott ($h/b=0.32$). The theoretical-sum mode characteristic nearly matches for the three cases and is not shown.

REFERENCES

[1] R. S. Elliott, "Two-mode waveguide for equal mode velocities," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-16, pp. 282–286, May 1968.

[2] P. Silvester, "A general high-order finite-element waveguide analysis program," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-17, pp. 204–210, Apr. 1969.

On the Solution of the Circularly Cylindrical Coordinate Wave Equation in Homogeneous Isotropic Regions Containing the Coordinate Axis

D. M. BOLLE

***Abstract*—In a number of well-known texts, misleading statements are made concerning the reason for eliminating the Neumann or Bessel function of the second kind from the solution of the wave equations in circular cylindrical coordinates for a homogeneous region containing the coordinate axis. This letter discusses the conditions that are required to arrive at a unique solution.**

In several texts generally consulted by students in the field of electromagnetic theory [1]–[5], erroneous and misleading statements are made concerning the reason for excluding the Bessel functions of the second kind from the solution of the wave equation in circular cylindrical coordinates in isotropic homogeneous regions containing the axis. The condition used is that the field quantities, in these cases E_Z or H_Z, must be bounded. Two objections to such a condition must be made. First, the statement is misleading in that students infer too readily that field components must always be bounded in such situations. Such is certainly not the case, e.g., in geometries where the boundaries exhibit sharp edges. Secondly, such a statement or condition does not rest on a demonstrable physical principle.

The finite energy condition, i.e., square integrability, can be invoked at this point and may be stated in the form

$$\int_0^{\delta V} |\psi|^2 dv \leq M.$$

Now, whenever higher order modes are considered where

$$\psi \propto [J_n(x), N_n(x)] \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} n\phi, \qquad n \neq 0$$

and

$$N_n(x) \sim x^{-n}(x \to 0), \qquad n = 1, 2, 3, \cdots$$

then the finite energy condition suffices to prohibit the use of the Neumann functions.

However, for regions with rotational symmetry where the lowest order admissable solution is

$$\psi \propto [J_0(x), N_0(x)]$$

and

$$N_0(x) \sim \log x(x \to 0)$$

there results

$$\int_0^{\delta x} x(\log x)^2 dx \leq M_0.$$

Hence, in this case, the finite energy condition is not sufficient.

This question is resolved only when, instead of restricting our attention to the axial field components, all the field components are examined. For the lowest order mode the following result is then obtained:

$$(H_r, E_r) \propto N_1(x) \sim x^{-1}(x \to 0)$$

and it is observed that, unless sources are present on the axis, such a solution is not acceptable.

Thus the crucial condition in rejection Bessel functions of the second kind is not boundedness nor finite energy, but that the region must be source free!

When the fractional-order Bessel functions arise, as is the case in reentrant sectorial cylindrical guides, then again the finite energy condition on the axial field components is not sufficient, but the radial components must be examined. The order of the singular behavior is such that Neumann solutions would indicate line sources on the edge of the reentrant sector. Thus, unless appropriate line sources are specified, such solutions must be eliminated.

A further related point should be considered. In problems involving cylindrical geometries, in particular the circular cylinder or coaxial circular cylinder, the solution of the wave equations yields trigonometric functions in the angular coordinate. If radial planes restrict the range of the angular coordinate, then the separation constant multiplying the angular coordinate in the argument of the trigonometric functions is determined explicitly by the boundary condition on the radial planes. However, if no such subdivision occurs, then, too often, the single-valuedness condition on the field quantities is used to obtain the undoubtedly correct result that the separation constant must be an integer. The single valuedness of the solution can only be used if the angular variable is allowed to range over $-\infty<\phi<\infty$, say, and such need not be the case since restricting the range of ϕ to $-\pi\leq\phi<\pi$, or $\phi_0\leq\phi<\phi_0+2\pi$, does not detract from the generality of the problem solution. In that event, single valuedness cannot be used, and the integral value of the separation constant must be arrived at through the condition that the field must be everywhere continuous. If any value other than an integral value is taken, a discontinuity in the field will occur at $\phi=\phi_0$, which would indicate the presence of a radial sheet of sources. It then follows that, if no

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such sources are specified, the separation constant must be assigned an integral value.

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REFERENCES

[1] J. Stratton, *Electromagnetic Theory.* New York: McGraw-Hill, 1941, p. 524.
[2] E. C. Jordan, *Electromagnetic Waves and Radiating Systems.* Englewood Cliffs, N. J.: Prentice-Hall, 1950, p. 275.
[3] R. E. Collin, *Field Theory of Guided Waves.* New York: McGraw-Hill, 1960, p. 195.
[4] J. D. Jackson, *Classical Electrodynamics.* New York: Wiley, 1962, p. 261.
[5] S. Ramo, J. R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics.* New York: Wiley, 1965, p. 429.

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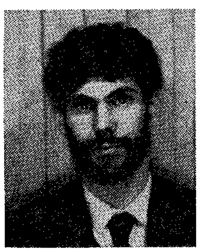

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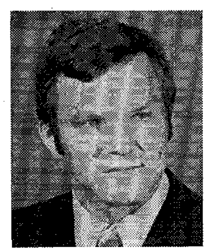

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